Abstract

Small denominator problems appear in various areas of analysis, PDE, and dynamical systems, including spectral theory of quasiperiodic Schrödinger operators, non-linear Schrödinger equations, and non-linear wave equations. These problems have traditionally been approached by KAM-type constructions. We will discuss the methods, originally developed in the spectral theory of quasiperiodic Schrödinger operators, that are both considerably simpler and lead to results unattainable through standard KAM techniques. For quasiperiodic operators, these methods have enabled precise treatment of various types of resonances and their combinations, leading to proofs of sharp (arithmetic) spectral transitions, the ten martini problem, and the discovery of universal hierarchical structures of eigenfunctions.