Abstract

A "drum" can be mathematically described by waves propagating on a Euclidean surface with boundary, which involves the Dirichlet Laplacian on this surface. Due to the linearity of the wave equation, one may focus on the stationary vibration modes, namely the eigenmodes of the Laplacian.

For a standard (circular) drum, those eigenmodes are known in great detail. On the other hand, if one modifies the shape of the drum, one has in general less information available.

Our objective is to understand the quantitative properties of these stationary modes, in particular in the high frequency regime, where waves can propagate along "rays" according go geometric optics. In this regime, the long time properties of the ray dynamics (or billiard dynamics) become relevant to understand the wave eigenmodes; these properties strongly depend on the shape of the drum. We will especially focus on drums for which the billiard dynamics is chaotic.

At the microscopic scale (scale of the fast oscillations), the eigenmodes feature intricate patterns (e.g. nodal patterns), which can be analyzed through phenomenological Random Wave models, which still lack rigorous justification.

Most rigorous results concern the fluctuations of the

eigenmodes at the macroscopic scale (scale of the drum): in the high frequency regime, most of the eigenmodes become equidistributed across the drum, a wave manifestation of the ergodicity of the billiard flow (one speaks of Quantum Ergodicity).

Does this equidistribution property hold down to some mesoscopic scales? Can some exceptional eigenmodes feature a different behaviour?

I will present some old and more recent results, and sketch the tools from semiclassical analysis used in the proofs.