Chern and the Mathematical Institute of Tohoku University

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Figure 1: Writer's Photo: Shiing-Shen Chern(left), Reiko Miyaoka(right) Shanghai, September 2001

1   Introduction

Shiing-Shen Chern (陳省身) (Fig.1) and É. Cartan (1869-1951) are geometers representing the 20th century. Chern became famous by his proof [2] of the Gauss-Bonnet theorem published in 1944、and since then was the world leader in differential geometry. The achievements and personality of Chern are described by Shoshichi Kobayashi in 「現代幾何学の流れ」(Current Flow of Modern Geometry Chern), 「数学セミナー」日本評論社, ("Sugakusemina" Nihonhyouronsha) 2003, and 「チャーン（陳省身）先生を偲んで」(In Memory of the Teacher Chern), which was published in 「数学通信」日本数学会 ("Sugaku Tushin" The Mathematical Society of Japan), Vol. 10, No. 3 (2005).

It is rarely written in these articles, but Chern and the Mathematical Institute of Tohoku University in Sendai, Japan are deeply related. In fact, much of Chern's data remain at the Institute. While introducing the poems that Chern wrote when he visited Tohoku University in Sendai, I will explain how I studied Chern's papers; and became a mathematician.

2   Shiing-Shen Chern and Shigeo Sasaki

Chern was born on October 26, 1911 in Zhejiang, China. At the age of 15, he entered Nankai University in Tianjin. He went on to graduate from the Tsinghua University graduate school in Beijing, and he later received a Ph.D. under Professor W. Blaschke of Hamburg University in Germany. Afterwards, he moved to Paris to continue the study of differential geometry under É. Cartan. In 1943, he moved to the United States and stayed at the Institute for Advanced Study in Princeton. Later, he taught at the University of Chicago and afterwards at the University of California Berkeley as a full professor. He later worked as the founding Director of the Mathematical Sciences Research Institute (MSRI) in California.

Chern had already written three papers before moving to Germany. Two of them were published by the Tohoku Mathematical Journal of Tohoku University and were the first of his articles published in a foreign journal. Later, Chern recalled that he was very happy when he received the letter from the editor-in-chief of Tohoku Mathematical Journal to accept these papers for publication. The Tohoku Mathematical Journal, established in August 1911, was the first mathematical journal in Western languages published in Japan. Chern's papers [1] were published in volume 40 of the Tohoku Mathematical Journal, in which 27 of the 35 authors were foreigners. Famous mathematicians such as J. Hadamard, J. Sz. Nagy, and Su Buqing contributed to this Journal.

My Ph.D. advisor, Shigeo Sasaki, was one year younger than Chern and was born on November 18, 1912 in Yamagata prefecture of Japan. He majored

in differential geometry at the Mathematical Institute of Tohoku Imperial University under Professor Tadahiko Kubota and was an associate professor at Tohoku Imperial University in 1944, and then became a full professor at Tohoku University in 1946.

Sasaki stayed at the Institute for Advanced Study in Princeton as a visiting professor from 1952 to 1954. On his way back to Japan, Chern invited Sasaki to spend two months at the University of Chicago. This was the beginning of the friendship between Chern and Sasaki. Professor Sasaki retired in March 1976 at the age of 63.

3 Chern's Lecture Notes

In June 1965 Chern first came to the Mathematical Institute of Tohoku University in Sendai, Japan by the invitation of Sasaki and the Ministry of Education of Japan. At that time, I was a fourth year student at the Faculty of Science, Tohoku University, and attended a lecture by Sasaki. I studied Chern's lecture notes " Differentiable Manifolds " 1959, University of Chicago under the guidance of Shukichi Tanno, who had just became a lecturer at Sasaki's laboratory. I learned the basics of how to read math books from Tanno, who is close to my age; and is like a brother to me. When Chern was invited to give a lecture at the colloquium of the Mathematical Institute of Tohoku University, I attended the colloquium just to see Chern in person. By the memory of Takashi Sakai, who worked at Ichinoseki National College of Technology at that time, there were two talks, one by Hsien-Chung Wang, whose subject was on Lie groups, and the other by Chern whose subject was the theory of value distribution of holomorphic functions. After the two talks, Tadao Tannaka, a famous professor in the algebraic number theory, said: "The main subject today was supposed to be geometry, but the talks were related to algebra and analysis". Chern quickly answered "Geometry covers all of mathematics." Then the venue got really excited. There is a signature book for invited speakers in the Colloquium of the Mathematical Institute. Figure 2 is the signature of Chern and the contents are as follows:
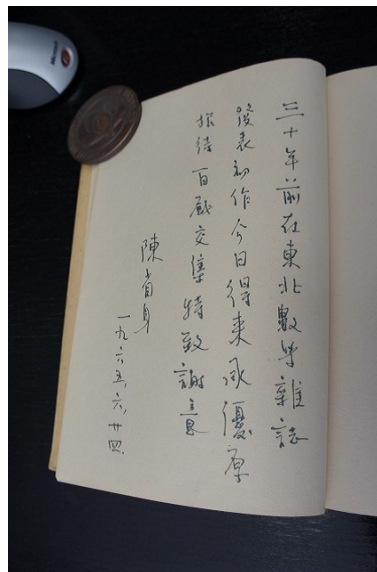

Figure 2: Sign of Chern, June 1965, Autograph book of the Colloquium of the Mathematical Institute, Tohoku University

"Thirty years ago, I was honored to have my first paper published in the Tohoku Mathematical Journal.

I appreciate the kind invitation.

I am happy to be here and thank you for the hospitality.

Shiing-Shen Chern June 24, 1965"

(Translated by Google with writer's modification)

I enrolled in the Graduate School of Tohoku University in April 1966, and took a lecture about complex manifolds by Sasaki. During the first lecture, Sasaki wrote the titles of several reference books on the board. One of them was the lecture notes " Complex Manifolds" Autumn 1955-Winter 1956, The University of Chicago by Chern. Immediately, I went to the library of the Mathematical Institute to borrow that book, but someone had already borrowed it. Then, I was advised that this book was in the bookshelf of Sasaki's office. Sasaki kindly lent me the book which was from Chern. Surprisingly it had a shabby binding and it included many notes by Sasaki.

I was excited just to read the book from my respected teacher, because now I knew how to study mathematics. I was fortunate to be a graduate student in Sasaki's laboratory. I made a copy of the lecture note without Sasaki's permission before returning. The copy is still in my hands and it is one of my treasures. However, I forgot to apologize to Sasaki before he died.

4 Chern's papers

The following year, I also studied Chern's work. On February 1967, when considering the theme of my master's thesis, Sasaki showed me several papers, including the joint-paper by Chern and Lashof for the total absolute curvature of submanifolds [3]. My master's thesis is a comprehensive report on the total absolute curvature of submanifolds, and there were no new results, but I was fortunate to become an assistant professor of Tohoku University. At that time, an assistant could be selected by the professor of each laboratory, and also since many science and engineering universities in Japan were opened, demand for mathematics teachers increased. In April 1968 I joined the civil service and became a research assistant at the Mathematical Institute of Tohoku University. I worked as the exercise assistant for Sasaki's lecture for third year students of the Mathematical Institute. In June of that year, Sasaki asked me to write an original paper and showed me a paper for Einstein complex hypersurfaces in a complex space form [4]. The topic Sasaki proposed to me was to consider the corresponding problem for the regular contact Riemannian manifolds[1]. I wrote several papers about this subject and one of them [6] was published in the Tohoku Mathematical Journal[2]. That paper's reception in the Sasaki's laboratory was negative and so, I thought it would be difficult to get a Ph.D. in this subject.

In the fall of 1970, Sasaki gave me an A4 sized book with a red cover and said: "This was sent by Chern to me, but I will give it to you". That was the lecture notes for Minimal Submanifolds by Chern at Kansas University[3]. I was already 28 years old, married and had a son. However, as a mathematician working towards a Ph.D., I thought that I had no choice but to read the book. In his lecture notes, Chern gave simple treatment using the theory of moving frames for the interesting results of minimal submanifolds proved in the 1960s such as Osserman's theorem on minimal surfaces in Euclidean space, and Simon's inequality on the minimal submanifolds in a sphere. I was very interested in those results. Next, I read in detail the paper by Chern for minimal spheres in the space form [5]. After that I succeeded in extending his method to any minimal surfaces in the space form and found minimal flat tori in the higher dimensional sphere. Based on those achievements, I got a Ph.D. from Tohoku University in 1975 under the supervision of Sasaki.

5 Chern's Visit to Sendai

Chern's second visit to Tohoku University was in August 1974, at the age of 63, and he was accompanied by his wife. Mr. and Mrs. Chern fully enjoyed their stay in Sendai as guests of Mr. and Mrs. Sasaki. Chern stated in the book "Shigeo Sasaki Selected Papers" (edited by S. Tachibana, Kinokuniya Company, 1985) that they were born at almost the same time, and the books they studied and their thoughts on geometry were also the same. Chern wrote a Chinese poem in the book. Here is the translation by Harry Wang:

" I presented my first writing,
Like trying a butcher's knife.
Upon growing older I came to realize,
For the first time learning's strife.
Having read my old paper,
After forty-one years,
It is lotus season,
Time to disseminate new knowledge.
With a common language and a common will,

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[1] They are called Sasakian manifolds

[2] Nowadays the manifold in this class is called a Kenmotsu manifold.

[3] "Minimal submanifolds in a Riemannian manifold" by S. S. Chern, University of Kansas, Department of Mathematics, Technical report **19**(New Series), 1968.

Figure 3: Writer's Photo: Hitoshi Takagi(sitting), Shukichi Tanno(left), Seiki Nishikawa, Shih-Ning Chern, Shiing-Shen Chern(right), in Matsushima, August 1974.

We search for truth and principles.
With oneness of mind and heart,
We seek the ultimate and the infinitesimal.
How can anyone say.
The mathematician's world is small.
Our joy is seeing our young elite,
Follow the footsteps of those before them.

August 1974   Shiing-Shen Chern"

One day during his stay in Sendai, when Tanno guided Mr. and Mrs. Chern to Matsushima, which is a famous sightseeing spot, my colleagues H. Takagi, S. Nishikawa and I joined them (Fig.3). I was nervous because it was my first opportunity to interact with a foreigner in a long time. I could not speak English well, so I did not remember what was said. Tanno died in 1999 at the age 62.

The next time Chern visited Japan was in 1977. In September of that year, the conference "The Japan-United States Seminar on Minimal Submanifolds, including Geodesics" was held in Tokyo. The person in charge of the Japanese side was T. Otsuki of the Tokyo Institute of Technology, and the US side was led by Chern. Fortunately, I was one of the invited speakers of this conference. That was a big deal for me later. In fact, the research exchange with the foreign mathematicians who attended this conference is still ongoing. In my presentation, I gave an extension of the Weierstrass formula for minimal surfaces to any surfaces [7]. Having a Ph.D. and looking for my next research theme, I remembered the advice of Sasaki that the time to solve both easy and difficult problems may not be so different. So, I tried to write another proof of the well-known Weierstrass-Enneper representation formula for minimal surfaces. I made the calculation of the mean curvature for a smooth surface in the Euclidean three space without using the minimality condition. As a result, I found a representation formula of any surface in the Euclidean three space that is not necessarily a minimal surface [7]. This result was announced at the 1977 "Japan-US Seminar" held in Tokyo.

6   Dedicated Paper

I never was a student of Chern, but I have a paper dedicated to him [8]. Why did I dedicate my paper to Chern? Let me explain. It was due to my stay in Cologne of Germany. On April 1978, by the research fellowship of the Alexander von Humboldt Foundation I visited Professor P. Dombrowski of the University of Cologne, Germany for a year. There, I met H. Reckziegel who was the associate professor of the department of Mathematics at the University of Cologne. It was in the fall of 1978 that Reckziegel was interested in drawing smooth curves in the plane by using the main computer of the computer center of the University of Cologne[4]. He told me, "The profile curve of the surface of revolution with mean curvature zero is a catenary, which is easy to draw by computer. What is the shape of the profile curve of the surface of revolution with non-zero constant mean curvature? I showed him a paper by Delaunay which classified surfaces of revolution with a constant mean curvature in the Euclidean three space. A few days later, Reckziegel told me, "The expression Delaunay discovered is unsuitable for drawing by com-

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[4]Please remember that there were no personal computers in 1978 and the first Macintosh 128K appeared on January 1984.
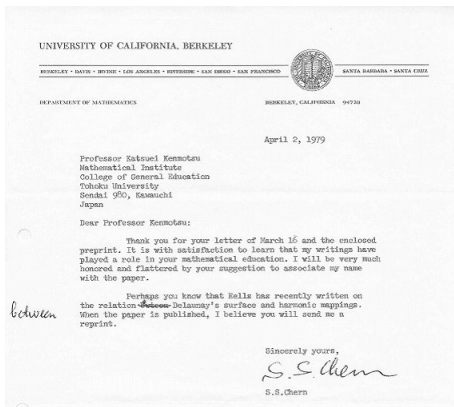
Figure 4: Chern's letter, dated April 2, 1979

Figure 5: Chern's lecture, October 1987, Mathematical Institute Tohoku University, Sendai Japan

puter. Since the improper integral is included in the formula, the computer calculation does not converge so fast." When I heard this, I intuitively thought that Delaunay's formula was not good. The reason is that Delaunay also showed in the same paper the geometric construction of the profile curve, in which there was no singularity. So, I believed that there was a different expression for the profile curve, and I was able to find a simpler formula [8]. More importantly, my calculation did not require the condition that the mean curvature of the surface of revolution be constant. In the end, I got a representation formula of the surface of revolution such that the mean curvature is the given one. Using that formula, Reckziegel and I drew the profile curve on the main computer at the University of Cologne. At that time, the computer center was drawing figures at night. So Recchigel and I punched the computer instructions in the morning, and we had lunch with Dombrowski at the university cafeteria. On the way back, we received the computer graphics that we requested the day before. No one knew what the shape of the curve would be, so it was exciting for us to look at the drawing.

Since I knew the importance of giving another proof of a famous result from reading Chern's papers and lecture notes, I was able to find a new proof for the Delaunay theorem. Also, I knew that the computer graphics posed a new problem for classical surfaces theory.

So I wanted to dedicate this paper [8] to Chern and wrote a letter to him asking if he would receive the dedication paper. I was very happy when I received a reply of consent from Chern (Fig. 4).

7   Chern's Last Stay in Sendai

Chern made his third visit to Sendai in October 1987 at the age of 76. He was one of the main invited speakers of the conference held in the Mathematical Institute of Tohoku University (Fig.5), and he also looked forward to seeing his old friend Sasaki. However, Sasaki died suddenly in August of that year at the age of 74.

Chern's poem (Fig.6), written in Chinese, describes his feelings at that time. Translation by Google with writer's modification is as follows:

" It happened to be made on October 26 1987
My life is always as a traveler,
So, I got one more year old this morning[5].
My friend has already passed away[6], and
I have a memory of my youth
in front of me[7].

Shiing-Shen Chern"

[5] I was born on October 26, 1911.

[6] Professor Shigeo Sasaki, who was an old friend, has passed away and I am full of sadness

[7] The first paper I wrote was published in the Tohoku Mathematical Journal.
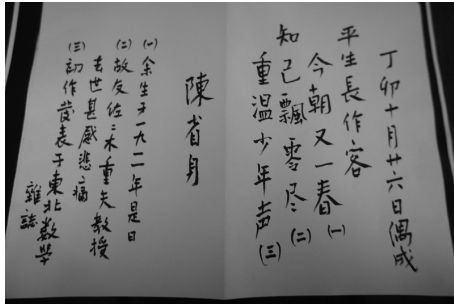
Figure 6: Chern's autograph poem and sign, October 26, 1987, Mathematical Institute Tohoku University, Sendai Japan

The last time I met Chern was in September 2001 at Fudan University in Shanghai, China, at the International Symposium on Differential Geometry in Honor of Professor Su Buqing on the centenary of his birth. Chern remembered me, and when I asked if I could take his picture, he leaned forward for my camera. That is the picture shown at the beginning in (Fig.1). On December 3, 2004, Chern died in Tianjin, China at the age 93.

8   Closing Remarks

I was impressed by Sasaki's third year lecture for the introduction to differential geometry at Tohoku University, and I wanted to become a mathematician studying geometry. Then, under the guidance of Sasaki as the Ph.D. advisor, I was able to study the work of Chern and follow the path of a mathematician. I read Chern's papers and lecture notes and learned the importance of finding another proof of a famous theorem. Such a proof sometimes changes the mathematical framework and creates a new world in mathematics. I also learned that a simple alternative proof moved researchers in unexpected directions.

I would like to thank Takashi Sakai (Professor Emeritus, Okayama University), Seiki Nishikawa (Professor Emeritus, Tohoku University), and Zhong Hua Hou (Professor, Dalian University of Technology) for writing this article.

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