Conference on
Poisson Geometry and Cluster Algebra

January 6-10, 2025

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Chern Institute of Mathematics, Nankai University

Schedule of Lectures

January 6 (Monday)

14:00	Registration	**Jiayuan Hotel**
15:00-17:30	Informal discussion and problem session	

January 7 (Tuesday)

Time	Speaker	Title	Chair
08:50-09:00	**Welcome Speech**		Shaoqiang Deng
09:00-09:45	Yunhe Sheng	Post-groups, post-groupoids and the Yang-Baxter equation	
09:45-10:00	**Tea Break**		
10:00-10:45	Jiefeng Liu	Differential calculus on Lie conformal algebroids	
10:45-11:00	**Tea Break**		
11:00-11:45	Yanpeng Li	Remarks on geometric multiplicities	
12:00	**Lunch**		
14:30-15:15	Long Guo	Schubert calculus and Schubert polynomials	Fang Li
15:15-15:30	**Tea Break**		
15:30-16:15	Yu Li	Degeneration of toric varieties into matroid Schubert varieties	
16:15-16:30	**Tea Break**		
16:30-17:15	Antoine de Saint Germain	Fixed points of DT transformations and exponents of Weyl groups	
17:30	**Dinner**		

January 8 (Wednesday)

Time	Speaker	Title	Chair
08:50-09:35	Jiang-Hua Lu	The standard Poisson structure on Schubert cells from the viewpoints of Poisson deformation and cluster structures	Yu Li
09:35-09:50	**Tea Break**		
09:50–10:35	Ana Balibanu	TBA	
10:35-11:00	**Group Photo and Tea Break**		
11:00-11:45	Michael Gekhtman	New generalized cluster structures on GL(n) - a case study	
12:00	**Lunch**		
14:30-15:15	Jiarui Fei	Crystal Structure of Upper Cluster Algebras	Deshou Zhong
15:15-15:30	**Tea Break**		
15:30-16:15	Xiaomeng Xu	Quantization of irregular Riemann-Hilbert maps	
16:15-16:30	**Tea Break**		
16:30-17:15	Yuancheng Xie	Full Kostant-Toda lattice and the Hessenberg varieties	
17:30	**Banquet**		

January 9 (Thursday)

Time	Speaker	Title	Chair
09:00-09:45	Fang Li	Poisson structures of a quantum cluster algebra and 2nd-stage quantization	Jiang-Hua Lu
09:45-10:00	**Tea Break**		
10:00–10:45	Dmytro Voloshyn	Poisson rational quasi-isomorphisms	
10:45-11:00	**Tea Break**		
11:00-11:45	Peigen Cao	Log-canonical cluster variables are compatible	
12:00	**Lunch**		
14:30-15:15	Min Huang	Non-commutative polygon groups and braid group realization	Ning Li
15:15-15:30	**Tea Break**		
15:30-16:15	Jinfeng Song	Poisson and cluster structures arising from quantum symmetric pairs	
16:15-16:30	**Tea Break**		
16:30-17:15	Boming Jia	Minimal nilpotent orbits of type D and E	
17:30	**Dinner**		

Title and Abstract

Log-canonical cluster variables are compatible

曹培根 (Peigen Cao)
University of Science of Technology of China

Abstract: A cluster algebra is a Z-subalgebra of a rational function field generated by a special set of generators called cluster variables, which are grouped into overlapping subsets of fixed size, called clusters. For a cluster algebra A of full rank, Genkhtman-Shapiro-Vainshtein proved that there exists a Poisson bracket on A such that any two cluster variables from the same cluster are log-canonical. In this talk, we show the converse result: if two cluster variables are log-canonical, then they are contained in the same cluster.

Crystal structure of upper cluster algebras

费佳睿(Jiarui Fei)
Shanghai Jiaotong University

Abstract: We describe the upper seminormal crystal structure for the μ-supported δ-vectors for any quiver with potential with reachable frozen vertices, or equivalently for the tropical points of the corresponding cluster $\mathcal{X}$-variety. We show that the crystal structure can be algebraically lifted to the generic basis of the upper cluster algebra. This can be viewed as an additive categorification of the crystal structure arising from cluster algebras. We introduce the biperfect bases in the cluster algebra setting and give a description of all biperfect bases.

New generalized cluster structures on $GL(n)$ - a case study

Michael Gekhtman
University of Notre Dame

Abstract:I will discuss a generalized cluster on $GL(n)$ compatible with the particular Poisson bracket that is homogeneous w.r.t. two-sided action of a Poisson-Lie group $G = GL(n) \times GL(n)$. Here the components of G are equipped with two "opposite" versions of the Cremmer-Gervais Poisson-Lie bracket. Our construction relies on birational Poisson maps that relate the Poisson homogeneous structure under investigation with the phase space of the finite Toda lattice and the Poisson dual of the Cremner-Gervais Poisson-Lie group. This is a joint work with M. Shapiro and A. Vainshtein.

Fixed points of DT transformations and exponents of Weyl groups

Antoine de Saint Germain (申瑞華)
The University of Hong Kong

Abstract: In this talk, we investigate fixed points of the DT transformation in a given cluster ensemble of finite type. We show that the DT admits a unique totally positive fixed point. From this, we define the cluster-exponents of the cluster ensemble, determine their values explicitly in terms of root systems, and use their values to obtain remarkable formulas concerning the number of clusters and cluster variables. This is based on joint work with Prof. Jiang-Hua Lu and Mr. Zihang Liu.

Schubert calculus and Schubert polynomials

郭龙 (Long Guo)
Nankai University

Abstract: The classical Schubert calculus is concerned with the ring structure of the cohomology of flag manifolds. More specifically, it studies the structure constants appearing in the expansion of the product of two Schubert basis elements. On the combinatorial side, Schubert polynomials serve as polynomial representatives of Schubert basis elements, and thereby the calculation of structure constants may be transformed into how to expand the product of two Schubert polynomials. In this talk, we shall give an overview of basic information around Schubert calculus and Schubert polynomials. We also build a combinatorial formula for the structure constants in the "separated" cases, extending (and providing a new proof of) a recent work of Knutson and Zinn-Justin.

Non-commutative polygon groups and braid group realization

黄敏(Min Huang)
Sun Yat-sen University (Zhu Hai)

Abstract: The aim of my talk (based on joint work in progress with Arkady Berenstein, Vladimir Retakh and Eugen Rogozinnikov) is to introduce a class of non-commutative groups associated to dissections of an orbifold and study their symmetries. We will also realize braid groups via the automorphisms of the polygon groups.

Minimal Nilpotent Orbits of type D and E

贾博名(Boming Jia)
Tsinghua University

Abstract: In this talk, I will first review and explain the well-known result that the closure of the minimal nilpotent orbit $\mathcal{O}_{min}$ in $so(2n)$ is isomorphic to the affinization of $T^*(SL(n-1)/[P,P])$ for certain parabolic subgroup P of $SL(n-1)$. Then I will explain a quantum version of the above result. Finally I will prove an analogous result for the minimal nilpotent orbit of type E_6. If time permits I will also explain how to generalize the result to E_7 and (maybe) to E_8.

Poisson structures of a quantum cluster algebra and 2nd-stage quantization

李方(Fang Li)
Zhejiang University

Abstract: Motivated by the phenomenon that compatible Poisson structures on a cluster algebra play a key role on its quantization (that is, quantum cluster algebra), we introduce the 2nd-stage quantization of a quantum cluster algebra, which means the correspondence between compatible Poisson structures of the quantum cluster algebra and its 2nd-stage quantized cluster algebras. Based on this observation, we find that a quantum cluster algebra possesses a mutually alternating quantum cluster algebra such that their 2nd-stage quantization can be essentially the same. As an example, we give the 2nd-stage quantized cluster algebra $A_{p,q}(SL(2))$ of $Fun_{\mathbb{C}}(SL_q(2))$ and show that it is a non-trivial 2nd-stage quantization, which may be realized as a parallel supplement to two parameters quantization of the general quantum group. As another example, we present a class of quantum cluster algebras with coefficients which possess a non-trivial 2nd-stage quantization. In particular we obtain a class of quantum cluster algebras from surfaces with coefficients which possess non-trivial 2nd-stage quantization. This is based on joint work with Jie Pan.

Remarks on geometric multiplicities

李彦鹏(Yanpeng Li)
Sichuan University

Abstract: In this talk, we explore relations between geometric multiplicities and various topics such as tensor product multiplicities, cactus groups, cluster algebras and multi-Horn problems.

Degeneration of toric varieties into matroid Schubert varieties

李昱(Yu Li)
University of Toronto

Abstract: For every rational central essential hyperplane arrangement $\mathcal{A}$, we construct a flat family of schemes over the affine line, such that the nonzero fibers are the toric variety associated with $\mathcal{A}$, and the zero fiber is an equidimensional scheme such that the reduced subscheme of each of its irreducible components is a matroid Schubert variety. The matroid Schubert variety associated with $\mathcal{A}$ is always an irreducible component of the zero fiber, and it is the only one when $\mathcal{A}$ is totally unimodular. We present a conjectural formula for the multiplicity of the irreducible components of the zero fiber in terms of the subarrangements of $\mathcal{A}$ of full rank. This is joint work in progress with Ana Balibanu and Colin Crowley.

Differential calculus on Lie conformal algebroids

刘杰锋(Jiefeng Liu)
Northeast Normal University

Abstract: In this talk, we first recall some constructions of Lie conformal algebroids. Then we introduce the representations, dual representations and cohomology of Lie conformal algebroids. Next the differential calculus on Lie algerbroids is studied. Finally, we give the isomorphism between the cohomology of Lie conformal algebroids and de Rham complex on algebra of differential functions

The standard Poisson structure on Schubert cells from the viewpoints of Poisson deformation and cluster structures

路江华(Jiang-Hua Lu)
The University of Hong Kong

Abstract: In this talk we show that the standard Poisson structure on a Schubert cell in the flag variety of a complex sem-simple Lie group is, in a sense, a master deformation of its log-canonical term, and we point out a mysterious relation between the cohomology classes appearing in the deformation and the mutation matrix for the standard cluster structure on the Schubert cell. This is joint work with Mykola Matviichuk.

Poisson and cluster structures arising from quantum symmetric pairs

宋金峰(Jinfeng Song)
National University of Singapore

Abstract: Given a semisimple Lie algebra $\mathfrak{g}$ and an involution θ of the algebra, the quantum symmetric pair is a quantization of the universal enveloping algebras of the pair $(\mathfrak{g}, \mathfrak{g}^\theta)$. It consists of the Drinfeld-Jimbo quantum group associated with $\mathfrak{g}$, and a coideal subalgebra algebra, called an i-quantum group. It is well-known that quantum groups are also quantized coordinate algebras of the dual Poisson-Lie groups. In this talk, I will explain that i-quantum groups are also quantized coordinate algebras of Poisson homogeneous spaces. When the symmetric pair is (sl_n, so_n), we obtain Dubrovin–Ugaglia Poisson structures on the space the unipotent upper triangular matrices. Motivated by the Chekhov-Shapiro' s log canonical coordinates, we construct an algebra embedding from the associated i-quantum group to a quantum cluster algebra.

Post-groups, post-groupoids and the Yang-Baxter equation

生云鹤(Yunhe Sheng)
Jilin University

Abstract: We introduce the notion of post-groups, which are the underlying structures of Rota-Baxter operators on groups. The differentiation of post-Lie groups gives post-Lie algebras. Post-groups are also related to braces and Lie-Butcher groups, and give rise to set-theoretical solutions of Yang-Baxter equations. We further introduce the notion of post-groupoids, whose differentiations are post-Lie algebroids. We show that post-groupoids give quiver-theoretical solutions of the Yang-Baxter equation on the underlying quiver of the subadjacent groupoids. The talk is based on the joint work with Chengming Bai, Li Guo, Rong Tang and Chenchang Zhu.

Poisson rational quasi-isomorphisms

Dmytro Voloshyn
Institute for Basic Science, ROK

Abstract: In recent years, the progress on cluster algebras compatible with Poisson brackets gave rise to Poisson rational quasi-isomorphisms. When such morphisms are birational, one can use them to construct log-canonical coordinates relative 'more complicated' Poisson brackets from the 'more simple' ones. In the context of Belavin-Drinfeld (BD) brackets, this means, in particular, that one can construct a log-canonical system for a nontrivial BD bracket if one has such a system for the trivial BD bracket.

I will explain the construction of such Poisson rational quasi-isomorphisms for groups, their Drinfeld doubles and Poisson duals, as well as some principles of constructing them for other Poisson varieties. I will also report on the current status of the GSV conjecture.

Full Kostant-Toda lattice and the Hessenberg varieties

谢远成(Yuancheng Xie)
Peking University

Abstract: Toda lattice is an integrable lattice model describing motions of a chain of particles with exponential interactions between nearest neighbors. Since 1967 after its discovery, Toda lattice and its generalizations have been the test models for various techniques and philosophies in integrable systems and wide connections are built with many other branches of mathematics. In this talk, I will talk about its connection with the so-called Hessenberg varieties among which Peterson variety and flag varieties are the most well-known ones. This talk is based on a joint work with Yuji Kodama.

Quantization of irregular Riemann-Hilbert maps

徐晓濛(Xiaomeng Xu)
Peking University

Abstract: This talk gives an introduction to the Poisson geometric nature of the irregular Riemann-Hilbert map. It then introduces its quantization and some relations with cluster algebras and crystal basis.