Abstract

In 1919, Besicovitch constructed a compact set in the plane with Lebesgue measure 0 that contains a unit line segment pointing in every direction. Such objects are now called measure 0 Besicovitch sets (aka Kakeya sets). By replacing a measure zero Besicovitch set by its \delta-thickening, one obtains a collection of 1 x \delta rectangles pointing in different directions, the sum of whose areas is 1, but whose union has very small volume. The existence of such collections of rectangles is called the Besicovitch compression phenomenon.

The Kakeya set conjecture is a quantitative statement controlling the strength of the Besicovitch compression phenomenon. In this talk, I will discuss connections between the Besicovitch compression phenomenon, the Kakeya set conjecture, and questions in harmonic analysis and PDE.