TRANSFORMATIONS OF HYPERGEOMETRIC MOTIVES

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Abstract. Transformation formulas for hypergeometric functions have been known for more than 200 years. For instance,

$$ {}_2F_1\left[\begin{matrix} a & b \\ & 2b \end{matrix}; z\right] = (1-z)^{-a/2}\,{}_2F_1\left[\begin{matrix} a/2 & (b-a)/2 \\ & b+1/2 \end{matrix}; \frac{z^2}{4z-4}\right] $$

due to Kummer. Today the term *hypergeometric* encompasses various generalizations, e.g., ${}_pF_q$, Pochhammer, Appell-Lauricella. The most general set-up is the GKZ (Gelfand-Kapranov-Zelevinski) formalism. Also there are hypergeometric functions over other fields, for instance, p-adic fields $\mathbb{Q}_p$, finite fields $\mathbb{F}_q$. Over a finite-field, a hypergeometric function is a character sum

$$ {}_2\mathbb{F}_1\left[\begin{matrix} A_0 & A_1 \\ & B_1 \end{matrix}; \lambda; q\right]. $$

This talk will survey some recent work on transformations of classical and finite-field hypergeometric functions. The key is that hypergeometric functions are *motivic*. that is, they arise from families of algebraic varieties. Especially there are *local systems*, complex local systems for classical hypergeometric functions, ℓ-adic local systems for finite-field hypergeometric functions. Transformation formulas can often be understood as transformations among the corresponding motives. This gives a unified viewpoint allowing one to prove transformation formulas simultaneously over $\mathbb{C}$, and over $\mathbb{F}_q$.

We discuss some recent formulas of Tu/Yang coming from the theory of automorphic forms on Shimura curves, and a cubic transformation of an Appell-Lauricella function discussed by Koike/Shiga and Frechette/Swisher/Tu. This is a joint project with Fang-Ting Tu.

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