Abstract

Quadratic Hénon maps are polynomial automorphism of $\mathbb{C}^2$ of the form $h:(x,y)\mapsto (\lambda^{1/2}(x^2+c)-\lambda y,x)$. They have constant Jacobian equal to λ and they admit two fixed points. If λ is on the unit circle (one says the map h is conservative) these fixed points can be elliptic or hyperbolic. In the elliptic case, a simple application of Siegel Theorem shows (under a Diophantine assumption) that h admits many quasi-periodic orbits with two frequencies in the neighborhood of its fixed points. Surprisingly, in some hyperbolic cases, S. Ushiki observed some years ago what seems to be quasi-periodic orbits though no Siegel disks exist. I will explain why this is the case. This theoretical framework also predicts and mathematically proves, in the dissipative case (λ of module less than 1), the existence of (attractive) Herman rings. These Herman rings, which were not observed before, can be produced in numerical experiments.