Cone spherical metrics and stable bundles

Bin Xu(USTC)

Abstract: *Cone spherical metrics* are conformal metrics with constant curvature one and finitely many conical singularities on compact Riemann surfaces. A cone spherical metric is called *irreducible* if each developing map of the metric does *not* have monodromy lying in $U(1)$. We establish on compact Riemann surfaces of positive genera a correspondence between irreducible cone spherical metrics with cone angles being integral multiples of 2π and line subbundles of rank two stable vector bundles. Then we are motivated by it to prove a theorem of Lange-type that there always exists a stable extension E of L^{-1} by L, for L being a line bundle of negative degree on each compact Riemann surface of genus greater than one. At last, as an application of these two results and the Hermitian-Einstein metric on E, we find a new class of irreducible spherical metrics with cone angles being integral multiples of 2π on a compact Riemann surface X of genus $g_X > 1$. In particular, given an effective divisor D on X with degree even and bigger than $2g_X - 2$, we show that there exists a non-empty open subset in the complete linear system $|D|$ such that each divisor in this open subset can be represented by an irreducible metric on X. This is my joint work with Xuemiao Chen, Lingguang Li and Jijian Song.