## Abstract

Let A and G be finite groups having coprime orders and suppose that A acts on G via automorphisms. We give some solvability criteria for G according to the number of orbits that appear by the action of the fixed point subgroup  $C_G(A)$  on the set of maximal A-invariant subgroups of G, and likewise, on the set of non-nilpotent maximal A-invariant subgroups. We also obtain some characterizations and further structure properties of these groups. In the course of our study we prove an independent result concerning maximal factorizations of classical simple groups.