

## Abstract

For the planar  $N$ -body problem, we introduce a class of moving frame suitable for orbits near central configurations, especially for total collision orbits. The moving frame allows us to reduce the degeneracy of the problem according to intrinsic symmetrical characteristic of the  $N$ -Body problem. First, we give a full answer to the infinite spin or Painlevé-Wintner problem in the case corresponding to nondegenerate central configurations. Then following some original ideas of C.L. Siegel, especially the idea of normal forms, and applying the theory of central manifolds, we give a partial answer to the problem in the case corresponding to degenerate central configurations. We completely answer the problem in the case corresponding to central configurations with degree of degeneracy one. Combining some results on the planar nonhyperbolic equilibrium point, we give a criterion for the case corresponding to central configurations with degrees of degeneracy two. We further answer the problem in the case corresponding to all known degenerate central configurations of four-body. Therefore, we solve the problem for almost every choice of the masses of the four-body problem. Finally, we give a measure of the set of initial conditions leading to total collisions.