Abstract

The uniformization theorem is one of the most significant results in mathematics, with a deceptively simple statement familiar to many: every simply connected Riemann surface is biholomorphic to one of three standard surfaces—the Riemann sphere, the complex plane, or the open unit disk. This theorem has profound extensions to higher dimensions, including Thurston's geometrization program and, notably, the resolution of the Poincaré conjecture. It is no wonder that this theorem is one of the most important theorems, if not the most important theorem, in the last 150 years.

However, its rich history and far-reaching impacts are often underappreciated. For example, the initial formulations and attempted proofs by Klein and Poincaré using the method of continuity introduced many original ideas that have not been fully explored. Later, Teichmüller's innovative use of the method of continuity played a crucial role in his revolutionary work on Teichmüller space, profoundly influencing the study of moduli spaces of Riemann surfaces. Yet, this deep historical and conceptual connection is frequently overlooked.

In this talk, I will trace the historical development of the uniformization theorem, from its early formulations to its

lasting influence across mathematics. I will explore its connections to seemingly unrelated results and modern mathematical areas, particularly in Teichmüller theory and moduli spaces. Through this exploration, I aim to shed light on the theorem’s profound influence, inspire new perspectives on its unifying role in contemporary mathematics.