

## **Abstract**

In the past two decades, since the discovery of the figure-8 orbit by Chenciner and Montgomery, the variational method has become one of the most popular tools for constructing new solutions of the N-body problem and its extended problems. However, finding solutions via variations to the restricted three-body problem, i.e. the three-body problem involving a colliding two-body system and a massless particle, remains a great difficulty. One of the major reasons is the essential differences between two-body and three-body collisions.

In this paper, we consider a similar three-body system with less difficulty, that is involving a massless particle and a collision Kepler system with one body fixed. It does not generally reduce to a classical three-body problem. However, by an in-depth analysis of the asymptotic behavior of the minimizer, and an argument of critical and inflection points, we prove the Sundman-Sperling estimates near the three-body collision for the minimizer. With these estimates, we prove a class of collision-free solutions with prescribed boundary angles. Finally, we proceed to construct the periodic or quasi-periodic solution for restricted three-body problem with the extended collision

Kepler system.